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An Experimental Investigation of the Compressible Turbulent Boundary Layer with a Favorable Pressure Gradient

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This paper describes the results of a detailed experimental investigation of a two-dimensional turbulent boundary layer in a favorable pressure gradient where the freestream Mach number varied from 3.8 to 4.6; the ratio of wall to adiabatic wall temperature remained constant at a value of 0.82. Detailed profile measurements were made with pressure and temperature probes; skin friction was measured directly with a shear balance. The velocity and temperature profile results are compared with zero pressure gradient and incompressible results. The skin-friction data are correlated with momentum-thickness Reynolds number and the pressure gradient parameter $\beta_\theta = (\theta/\tau_w)(dP/dx)$. The skin friction increases with decreasing β_θ for a constant value of momentum-thickness Reynolds number.

Nomenclature

A = constant in Eq. (4)
B = constant in Eq. (4)
C_f = skin-friction coefficient
H_u = incompressible shape factor
k = Karman's constant
M = Mach number
N = velocity profile exponent $u/u_e = (y/\delta)^{1/N}$
P = pressure
Re_θ = momentum-thickness Reynolds number
T = temperature
$\bar{T}$ = $(T_t - T_w)/(T_{te} - T_w)$
u = velocity
u_τ = shear velocity = $(\tau_w/\rho_w)^{1/2}$
u^+ = u/u_τ
x = distance along plate
y = distance normal to flow
y^+ = $u_\tau y/\nu_w$
β = Clauser's equilibrium parameter = $(\delta^*/\tau_w)(dP/dx)$
β_θ = pressure gradient parameter = $(\theta/\tau_w)(dP/dx)$
δ = boundary-layer thickness
ρ = density
δ^* = displacement thickness
θ = momentum thickness
τ = shear stress
ν = kinematic viscosity

Subscripts

aw = adiabatic wall
e = freestream conditions
o = supply conditions
w = wall conditions
t = total conditions

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Introduction

THE work presented herein is part of an experimental program being carried out at the U. S. Naval Ordnance Laboratory (NOL) in which the turbulent boundary-layer flow is studied systematically and in detail with conventional pressure and temperature probes, and with a skin-friction balance. The earlier studies at zero pressure gradient and moderate heat-transfer rates were reported in Ref. 1. The present paper describes the results of the study of the turbulent boundary layer on the flat plate of the NOL Boundary-Layer Channel[2] along which the freestream Mach number varied from 3.8 to 4.6, and for moderate heat-transfer rates. Typical velocity and temperature profile data are presented along with skin-friction coefficients measured with a friction balance. The effect of the favorable pressure gradient on the boundary-layer flow structure and friction drag is discussed, and compared with supersonic zero pressure gradient and incompressible flow.

Nozzle Design

The main component of the NOL Boundary-Layer Channel is the two-dimensional supersonic half-nozzle shown in Fig. 1. One wall of the nozzle is a flat plate, 8 ft long and 12 in. wide. The opposite wall is a flexible plate which may be adjusted to give a prescribed Mach number distribution along the flat plate. Because of the numerous pressure gradient models

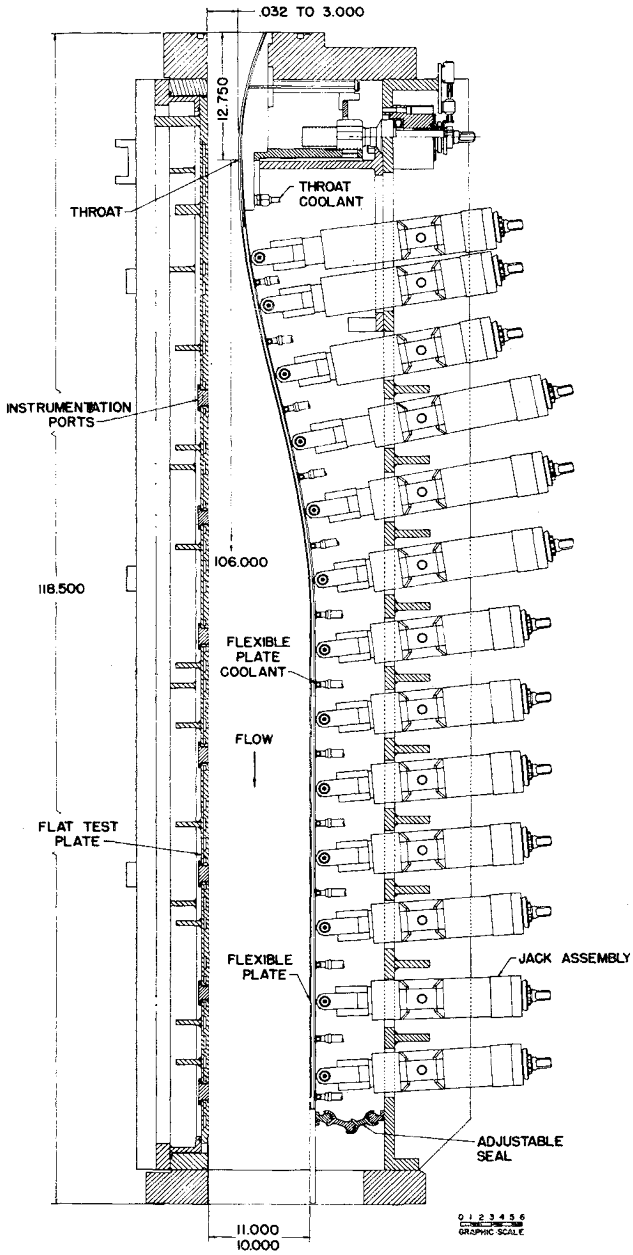


Fig. 1 Boundary-layer channel flexible nozzle.

that the flexible plate nozzle can provide considerable effort was devoted in the initial phase of the project to arrive at a meaningful pressure gradient model. Three approaches were investigated: 1) maintain constant value of H_u along the plate (this makes use of the incompressible flow concept of von Doenhoff and Tetervin[3] relating the shape factor to pressure gradient flows), 2) maintain du_e/dx constant along the plate, and 3) maintain constant value of β along the plate. (This follows the incompressible flow equilibrium boundary-layer concept of Clauser.[4]) The latter approach was more readily adapted to the nozzle configuration and consequently was used in this investigation.

Details of the Experiment

The experiments were performed in the NOL Boundary-Layer Channel at tunnel supply pressures between 1 and 10 atm, and a tunnel supply temperature of 150°C. The flexible wall of the tunnel was adjusted in the present test to give a prescribed Mach number distribution described previously. The actual Mach number distribution is shown in Fig. 2. The impact probe survey, shown in Fig. 3, along the

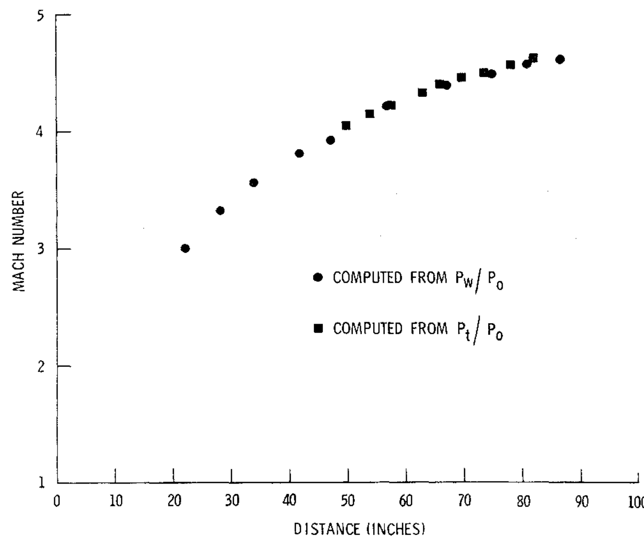


Fig. 2 Mach number variation along flat plate.

flat plate and 3 in. from the plate, indicates that the flow was shock free. The momentum-thickness Reynolds number varied in the present test from 7500 to 48,000; the ratio of wall to adiabatic wall temperature was constant at 0.82. Further details of the channel and its performance are given in Ref. 2. Measurements were taken at five stations along the test plate; at 50, 60, 70, 78, and 84 in. from the nozzle throat. Typical boundary-layer thicknesses along the test section ranged from 1.3 in. to 3.0 in.

Instrumentation

The boundary-layer profile surveys were made by traversing a Pitot pressure probe and an equilibrium conical temperature probe across the boundary layer. Both probes were mounted in a single holder, which was designed and tested so that there was not any probe interference between the two probes. The probes were traversed from the freestream toward the plate, with maximum movement of 3 in. The traverse was stopped at each point in the boundary layer at which data were taken, and the temperature and pressure were allowed to reach equilibrium.

The profile data were recorded automatically on NOL's PADRE.[5] This unit provides seven channels with servo-systems and direct digital conversion to permit recording the data directly on IBM cards.

Pitot pressure probes were made of 0.125-in.-diam stainless steel tubing flattened at the tip to a rectangular cross section of 0.005 × 0.100 in. (outside dimensions). The wall static pressure was measured by the 0.032-in. i.d. orifices in the flat plate. The local Mach number was computed from the Rayleigh-Pitot tube formula using the measured Pitot and wall static pressure.

The basic design of the equilibrium conical temperature probe is described in Ref. 6. Essentially the equilibrium temperature of a sharp 10° platinum cone was measured by a thermocouple mounted into its 0.050-in.-diam base. The measured cone temperature together with the measured local Mach number and cone tables provided the necessary information to calculate the local stagnation and static temperatures. In Ref. 6, the recovery factor for the cone was calibrated in the freestream to be that of the laminar flow value and this value was maintained constant across the boundary layer. In the present application, a cone recovery factor equal to the square root of the Prandtl number was assumed in order to compensate for cone temperature variations beyond those anticipated in Ref. 6. The assumption of a laminar cone recovery factor is based on the low cone length Reynolds number even though the flow about the cone is turbulent.

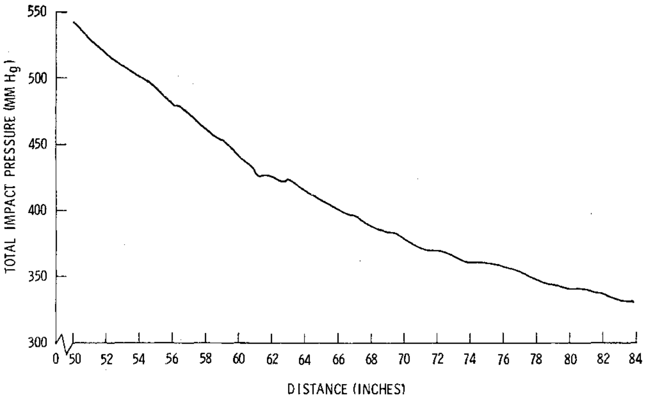


Fig. 3 Impact pressure probe survey along flat plate.

The results from the fine-wire stagnation temperature probe support this assumption.

A fine-wire stagnation temperature probe was also used to measure the temperature profile through the boundary layer. The basic features of this probe are described in Ref. 7. The probe consists of a chromel-alumel thermocouple wire placed normal to the flow. Using the measured Mach number distribution and conventional empirical equations for predicting the heat losses to and from the probe, the local stagnation temperature and hence the local static temperature may be obtained.

The static pressure profile across the boundary layer was measured with a 0.125-in.-o.d. static pressure probe. The tip of the probe was a conical 10° cone, and the static orifices were located on the circular tubing 20 diameters from the tip.

The local Mach number distribution was computed from the measured Mach number and temperature distributions. The edge of the boundary layer was selected as the location where the velocity gradient became zero.

The NOL skin-friction balance is described in Ref. 8. The instrument measures directly the shear drag on a 0.5-in.^{2} surface floating element mounted flush with the flat plate. The balance was designed for measurements in flows with heat transfer and pressure gradient.

Experimental Results

The Mach number distribution along the flat plate (see Fig. 2) was computed from the ratio of wall static to tunnel supply pressure (see Fig. 3). Both methods agree within 2%, indicating that the static pressure is constant across the boundary layer. To further verify this result, static pressure surveys were taken through the boundary layer. The typical results shown in Fig. 4 indicate that the static pressure is practically constant across the boundary layer.

It is well known that the cross-stream pressure gradients on the nozzle side walls of conventional, symmetrical, two-dimensional, uniform-flow nozzles can have large effects on the development of the nozzle wall boundary layers. The two dimensionality of the flat-plate boundary layer was investigated by two supplement experiments namely by oil flow studies on the flat-plate surface and by additional velocity profile measurements 3 in. to each side of the plate centerline at selected distances from the nozzle throat. The oil-flow studies showed the surface streamline pattern to be parallel in the center 8-in. region of the plate. The off-centerline velocity profile measurements compare with the centerline measurements within the measured nonuniformity of the free-stream flow, i.e., $\pm 0.75\%$. It is concluded from these tests that the flow is two-dimensional within the center 6 in. of the flat plate.

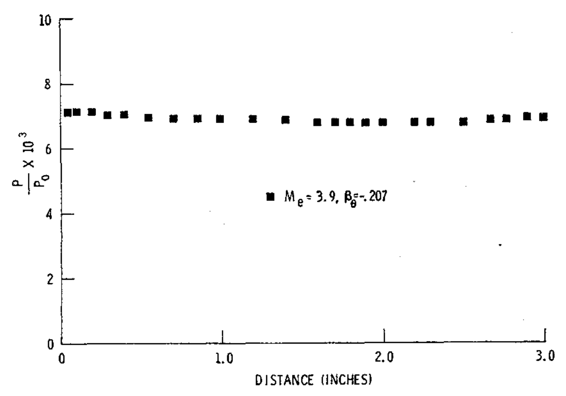


Fig. 4 Static pressure distribution through the boundary layer.

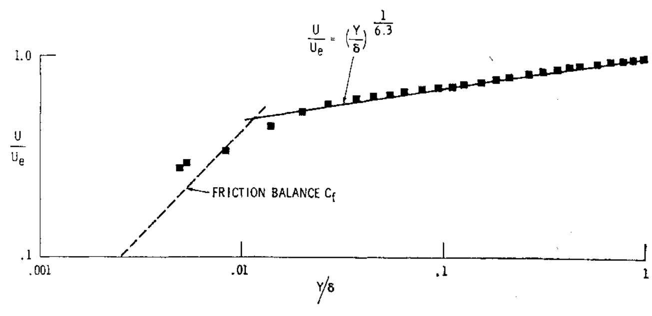


Fig. 5 Velocity profile $Re_\theta = 8290$, $M_e = 3.87$, $\beta_\theta = -0.207$.

A typical boundary-layer velocity profile is shown in Fig. 5. The tabulated data will be reported in Ref. 9. The line drawn in the laminar sublayer region was computed from the friction balance measurement. The fact that the last two velocity profile points do not fall along the curve computed from the friction balance measurement is possibly due to probe-wall interference. In the case of pressure gradient flow, a power profile was a better fit to the outer region of the boundary layer than in the zero pressure gradient case.[1] A least squares fit of the power profile exponent vs momentum thickness Reynolds number shows a value of the exponent slightly higher than for the zero pressure gradient flow case (see Fig. 6).

The correlation between the shape factor H_u and momentum-thickness Reynolds number is shown in Fig. 7. The favorable pressure gradient data is lower than the zero pressure gradient data, and has the same general variation with momentum-thickness Reynolds number. This is consistent with incompressible results.[10]

As shown in Ref. 11 a favorable pressure gradient on the temperature-velocity profile is to displace the curve from the zero-pressure-gradient results of Crocco. A typical tempera-

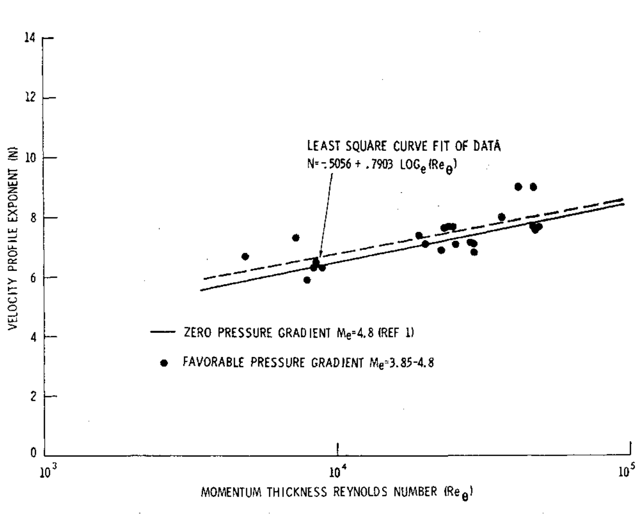


Fig. 6 Variation of velocity profile exponent with momentum thickness Reynolds number for $T_w/T_{aw} = 0.82$.

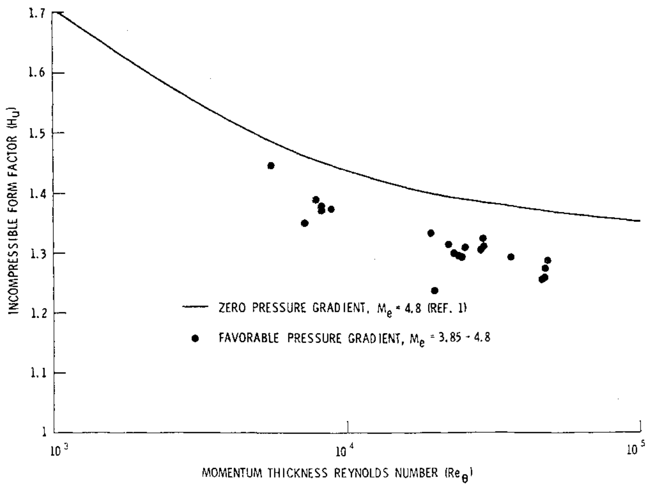


Fig. 7 Variation of incompressible form factor with momentum thickness Reynolds number for $T_w/T_{aw} = 0.82$.

ture-velocity correlation is shown in Fig. 8. Shown for comparison is Crocco's equation[10]

$$\bar{T} = u/u_e \tag{1}$$

and Walz's equation[12] for zero heat transfer

$$\bar{T} = (u/u_e)^2 \tag{2}$$

The data in the sublayer regions follow the zero pressure gradient adiabatic flow relation of Walz, and is fuller in the outer turbulent region. The displacement of the data from Crocco's equation is consistent with that predicted in Ref. 11. An inflection point in the temperature-velocity profile occurring at the lower edge of the logarithmic region of the boundary layer was observed consistently in all the data. For the data shown in Fig. 8 this point is located at $u/u_e \approx 0.6$. Temperature profiles were measured with a fine-wire stagnation temperature probe at the same supply conditions. These results, shown in Fig. 8, exhibit the same trends as those of the conical probe. A most likely cause of this discontinuity is the viscous effect on the Pitot probe. The Reynolds number for the data points near the wall, based on probe height, is in the order of two, well into the slip flow regime.[13] A slip flow correction will shift these data points near the wall to lower velocity ratios.[14] The slip flow correction is not included in the data of the present paper.

The zero-pressure-gradient flow data reported in Ref. 1 also followed the quadratic trend of the favorable-pressure-gradient results in the total temperature-velocity correlation. These results are in agreement with general nozzle wall data.[15,16] As indicated by Bushnell[16] the correlation appeared to be independent of β_θ because of the close proximity of the zero-pressure-gradient flow studies to the upstream favorable-pressure-gradient flow of the nozzle. R. A. Jones[17] suggested that the quadratic trend may be due to temperature deficiencies near the side walls of the settling chamber. Preliminary experimental studies in which the settling chamber wall temperature was made equal to the supply temperature indicated no change in the quadratic trend. However, reducing the energy loss from the boundary layer at the throat by increasing the local plate temperature produced significant changes in the temperature-velocity relationship. This relationship exhibited a shift from the quadratic toward the linear (Crocco) form as the heat transfer at the throat was decreased. One can infer that the heat transfer at the throat has a strong influence on the temperature-velocity correlations.

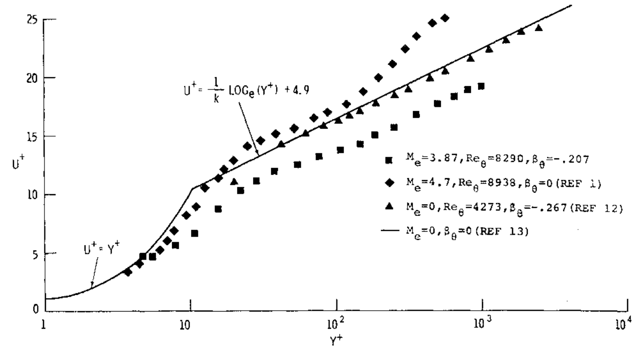


Fig. 9 Correlation of experimental results in terms of law of the wall.

Correlation of the favorable pressure gradient results in terms of the law-of-the-wall and velocity-defect law is shown in Figs. 9 and 10. Supersonic zero pressure gradient and incompressible results are shown for comparison.[18] The favorable pressure gradient data for $M_e = 3.87$ are seen to be below the supersonic pressure gradient results from $M_e = 4.7$, and the incompressible results for $\beta_\theta = -0.267$ and for $\beta_\theta = 0$. The results shown in Figs. 9 and 10 indicate that as a result of the favorable pressure gradient, the logarithmic region of the boundary layer has increased substantially in thickness and the velocity defect in the outer part of the layer has decreased. For incompressible flow the logarithmic portion of the boundary layer can be fitted by the equation[19]

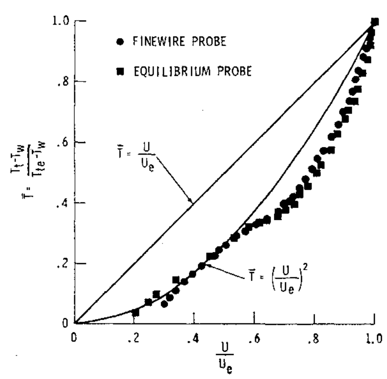
$$u^+ = (1/k)\ \ln y^+ + 4.9 \tag{3}$$

where $k = 0.4$ is Karman's constant. In the present investigation, it was observed that the logarithmic portion of the boundary layer could be fitted by the equation

$$u^+ = (1/A)\ \ln y^+ + B \tag{4}$$

where $A = 0.541$. B was not constant in the investigation.

Fig. 8 Temperature-velocity distribution for $T_w/T_{aw} = 0.82$, $Re_\theta = 8290$, $M_e = 3.87$, $\beta_\theta = 0.207$.



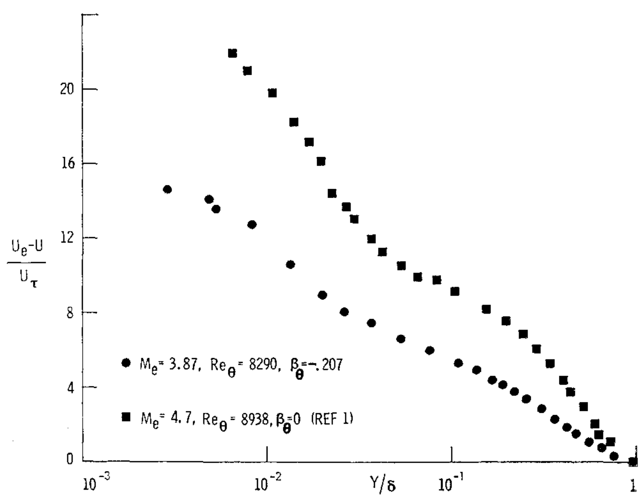


Fig. 10 Correlation of experimental results in terms of velocity defect law.

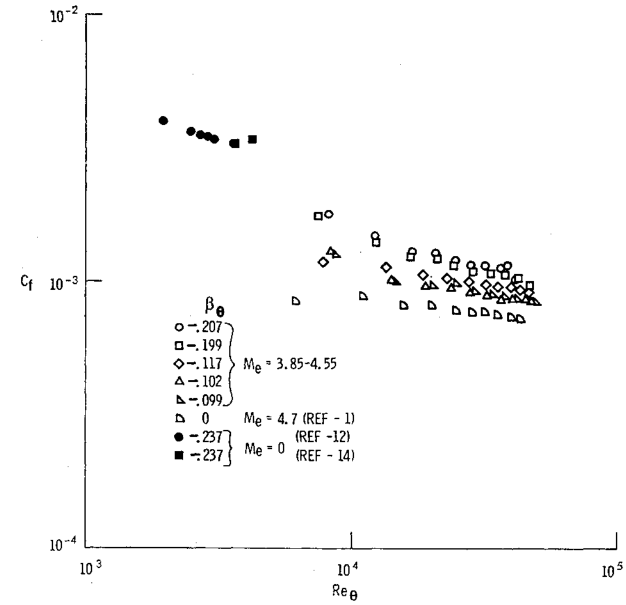


Fig. 11 Skin-friction correlation with momentum thickness Reynolds number.

The skin friction, obtained by a shear balance, is shown plotted against momentum-thickness Reynolds number in Fig. 11. Presented for comparison are the incompressible favorable-pressure-gradient results of Ludwieg-Tillman[18] and Herring and Norbury.[20] The present supersonic data are seen to lie well below the incompressible results. A consistent correlation of the present results in terms of the shape factor H_u was not observed. These results do not agree with the incompressible results of Ludwieg-Tillman[18] who were able to correlate the skin friction in terms of the shape factor. The measured skin friction in the present investigation was approximately 20% below that predicted by the combination of the Ludwieg-Tillman skin-friction formula and the reference enthalpy method.

A consistent trend in the present skin-friction data was observed when correlated in terms of the pressure gradient parameter $\beta_\theta = (\theta/\tau_w)(dP/dx)$. For increasing β_θ and constant Reynolds number, the skin friction decreases. The correlation of the skin friction in terms of β_θ is consistent with the incompressible results first proposed by Buri[10] who found similar results.

Conclusions

The effect of a favorable pressure gradient on supersonic turbulent boundary layers was studied in the NOL Boundary-Layer Channel, for $7500 < Re_\theta < 48{,}000$ and for $T_w/T_{aw} = 0.82$, with temperature and pressure probes and a shear balance.

The structure of the turbulent boundary layer was examined in terms of velocity and temperature profiles, law-of-the-wall, velocity defect law and incompressible form factor. The outer region of the boundary layer can be fitted with a power profile. The shape factor H_u decreases in a supersonic favorable pressure gradient. An inflection point in the temperature-velocity profile was observed to occur at the lower edge of the logarithmic portion of the boundary layer. The effect of a favorable pressure gradient is to thicken the logarithmic portion of the boundary layer and decrease the outer velocity defect portion.

The effect of a favorable pressure gradient on the turbulent skin friction was correlated in terms of the pressure gradient parameter β_θ. For increasing β_θ and constant Reynolds number, the skin friction decreases.

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